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LETTER TO THE EDITOR

Violation of universality induced by critical dynamics

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Abstract. The mapping of critical dynamics onto a static $(D+1)$-dimensional system is analysed in the large-n limit. At criticality Lifshitz triciritical behaviour (with upper critical dimensionality $D^*=4$) occurs in the neighbourhood of an ordinary Lifshitz point (with $D^*=6$). The violation of universality is a consequence of the very special path along which the critical point is approached.

Universality classes in the theory of critical phenomena are well understood. According to renormalisation group concepts, the critical manifold of a system is partitioned into basins of attraction around fixed points, having specific critical behaviour. Thus, the observed critical properties depend on the domain of attraction to which the system is driven, as the accessible parameters are tuned to criticality. This is *generically* true. Here we present an interesting exception, where the critical behaviour *differs* from the one associated with the domain of attraction of the fixed point. Of course, our result does not invalidate the well established scheme for universality illustrated above; rather it is the outcome of a *highly non-generic* way of approaching the critical point, and as such it is a novel critical phenomenon worth investigating.

This peculiar situation arises when critical dynamics in D dimensions is embedded into a static theory in $D+1$ dimensions (Peschel and Emery 1981, Rujan 1982a, b, Domany 1984, Schnieder *et al* 1985, Aharony *et al* 1985). The static image of the dynamical theory is obtained by constraining the parameters of a quite general class of underlying $(D+1)$-dimensional theories, which exhibit a Lifshitz tricritical point (LTP).

In this letter we reconsider the problem in the context of the large-n limit. The exact solution allows the nature of the constraint, needed to describe the dynamics, to be analysed in detail and to disclose a phenomenon neglected in previous studies. Although the asymptotic critical behaviour indeed belongs to the LTP universality class, the actual critical point turns out to be an ordinary Lifshitz point violating universality in the sense that the upper critical dimensionality is $D^*=4$ rather than $D^*=6$, as it should be for Lifshitz points (Hornreich *et al* 1975).

We consider a purely relaxational dynamical model of the Langevin type, without any conservation law:

$$\frac{\partial \boldsymbol{\phi}(\boldsymbol{x}, t)}{\partial t} = -\Gamma[-\nabla^2 + t(\phi^2)]\boldsymbol{\phi}(\boldsymbol{x}, t) + \boldsymbol{\eta}(\boldsymbol{x}, t) \tag{1}$$

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where $\boldsymbol{\phi}(\boldsymbol{x}, t) = (\phi_1(\boldsymbol{x}, t), \ldots, \phi_n(\boldsymbol{x}, t))$ is an n-component order parameter. The local interaction $t(\phi^2)((\phi^2)^2$ theory) is given by

$$t(\phi^2) = r_0 + (u_0/n)\phi^2 \tag{2}$$

and $\boldsymbol{\eta}(\boldsymbol{x}, t)$ is a Gaussian white noise with expectations

$$\langle \boldsymbol{\eta}(\boldsymbol{x}, t)\rangle = 0 \qquad \langle \eta_\alpha(\boldsymbol{x}, t)\eta_\beta(\boldsymbol{x}', t')\rangle = 2\Gamma T\delta_{\alpha\beta}\delta(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t') \tag{3}$$

with Γ being a kinetic coefficient.

All the dynamical information can be obtained from the probability measure associated with (1)

$$\mu[\boldsymbol{\phi}(\boldsymbol{x}, t)] \sim \exp(-S[\boldsymbol{\phi}(\boldsymbol{x}, t)]) \tag{4}$$

with the action functional $S[\boldsymbol{\phi}(\boldsymbol{x}, t)]$ in the large-n limit given by

$$S[\phi] = \int_{-\infty}^{\infty} \int \mathrm{d}^D x \left[\frac{1}{2\sigma}\left(\frac{\partial \boldsymbol{\phi}}{\partial t}\right)^2 + \frac{\sigma}{8}[(\nabla^2\boldsymbol{\phi})^2 - 2t(N)(\boldsymbol{\phi}\cdot\nabla^2\boldsymbol{\phi}) + t^2(N)\phi^2 \right.$$
$$\left. + 2(Q + t(N)N - n\delta^D(0))t'(N)\phi^2]\right] \tag{5}$$

where $N = \langle \phi^2 \rangle$ and $Q = -\langle \boldsymbol{\phi}\cdot\nabla^2\boldsymbol{\phi}\rangle$ are averages to be determined self-consistently. The origin of the divergent term $\delta^D(0)$ is discussed in the literature (Muñoz Sudupe and Alvarez-Estrada 1983, Zinn-Justin 1986). In the following it plays an important role.

The action functional (5) belongs to a class of functionals of the general form

$$\tilde{S}[\boldsymbol{\phi}(\boldsymbol{x}, t)] = \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}^D x \left[\frac{a}{2}\left(\frac{\partial \boldsymbol{\phi}}{\partial t}\right)^2 + \frac{b}{2}[(\nabla^2\boldsymbol{\phi})^2 - F(\phi^2)(\boldsymbol{\phi}\cdot\nabla^2\boldsymbol{\phi}) + G(\phi^2)]\right] \tag{6}$$

where $F(\cdot)$ and $G(\cdot)$ are arbitrary functions. The time variable can be treated as an additional space variable. Thus, $\tilde{S}[\boldsymbol{\phi}]$ corresponds to a free energy functional of a $(D+1)$-dimensional equilibrium system. The special form (5) of the action is obtained from (6) by imposing the constraints

$$4ab = 1 \qquad F(N) = 2t(N) \qquad G'(N) = \tfrac{1}{4}F^2(N) + \tfrac{1}{2}F(N)F'(N)N - nF'(N)\delta^D(0). \tag{7}$$

The problem we are addressing is the characterisation of the critical properties of the free energy functional (5) within the framework of the more general model defined by $\tilde{S}[\boldsymbol{\phi}]$.

In the large-n limit, the Fourier transform of the inverse propagator associated with (6) is given by

$$\tilde{C}^{-1}(\boldsymbol{q}, \omega) = a\omega^2 + bq^4 + r_{\mathrm{L}}q^2 + r \tag{8}$$

where

$$r_{\mathrm{L}} = bF(N) \tag{9}$$

$$r = b[QF'(N) + G'(N)]. \tag{10}$$

The critical surface in the parameter space is the manifold over which the inverse susceptibility r vanishes. Specialising it to a theory of the form

$$F(\phi^2) = f_1 + f_2(\phi^2) \qquad G(\phi^2) = g_1\phi^2 + \frac{g_2}{2}(\phi^2)^2 + \frac{g_3}{3}(\phi^2)^3 \tag{11}$$

it is convenient to regard g_1 as the parameter controlling deviations from criticality. Using (10), the critical value of g_1 is given by

$$g_{1c} = -f_2 Q_c - g_2 N_c - g_3 N_c^2 \tag{12}$$

with

$$N_c = \int_\omega \int_q \frac{1}{a\omega^2 + bq^4 + r_L(N_c)q^2} \qquad Q_c = \int_\omega \int_q \frac{q^2}{a\omega^2 + bq^4 + r_L(N_c)q^2}. \tag{13}$$

Once the critical surface has been located, it is straightforward to rewrite r_L and r in terms of the scaling fields (Zannetti and Di Castro 1977)

$$\begin{aligned} r_L &= \mu_{1L} + \mu_{2L}(N - N_c) \\ r &= \mu_1 + \mu_{2L}(Q - Q_c) + \mu_2(N - N_c) + \mu_3(N - N_c)^2 \end{aligned} \tag{14}$$

with

$$\begin{aligned} &\mu_{1L} = b(f_1 + f_2 N_c) \qquad \mu_{2L} = bf_2 \\ &\mu_1 = b(g_1 - g_{1c}) \qquad \mu_2 = b(g_2 + 3g_3 N_c) \qquad \mu_3 = bg_3. \end{aligned} \tag{15}$$

In this notation, the critical surface corresponds to $\mu_1 = 0$. Lifshitz critical behaviour is obtained when $r_L = 0$ and tricritical behaviour occurs for $\mu_2 = 0$ (Schneider *et al* 1985). The region on the critical surface relevant for critical dynamics is the line of Lifshitz points ($r_L = 0$, $\mu_2 > 0$) and the LTP ($r_L = 0$, $\mu_2 = 0$). The corresponding critical properties are obtained by evaluating the quantities $(N - N_c)$ and $(Q - Q_c)$ in terms of r and extracting the asymptotic solution from (14) (Schneider *et al* 1985, Aharony *et al* 1985). In particular, one finds that the upper critical dimensionality for the Lifshitz point is $D^* = 6$, while for LTP it is $D^* = 4$. Since the theory describing critical dynamics is defined by the action functional (5) with t given by (2), one expects that constraint (7) would lead to an LTP with $D^* = 4$. Actually, it is not quite so. Using constraint (7) the scaling fields can be rewritten as

$$\begin{aligned} &\mu_{1L} = 2b(r_0 + u_0 N_c) = b(f_1 + f_2 N_c) \qquad \mu_{2L} = 2bu_0 = bf_2 \\ &\mu_1 = \frac{1}{4b}(\mu_{1L}^2 + 2\mu_{1L}\mu_{2L}N_c) \qquad \mu_2 = \frac{1}{b}\mu_{1L}\mu_{2L} + \frac{1}{2b}\mu_{2L}^2 N_c \qquad \mu_3 = \frac{3}{4b}\mu_{2L}^2 \end{aligned} \tag{16}$$

and since in the original theory r_0 is the only free parameter, (16) defines a trajectory in the space of scaling fields which is tangential to the critical surface ($\mu_1 = 0$) for $\mu_{1L} = 0$. Furthermore, since this implies $r_L = 0$ and $\mu_2 = (1/2b)\mu_{2L}^2 N_c > 0$, the critical point associated with the trajectory (16) is an ordinary Lifshitz point, with $D^* = 6$.

However, along the trajectory, where the theory is governed by the action functional (5), the cancellation

$$Q + t(N)N - n\delta^D(0) = 0 \tag{17}$$

occurs self-consistently, reducing the inverse susceptibility to the simplified expression

$$r = \frac{1}{4b}[\mu_{1L}^2 + 2\mu_{1L}\mu_{2L}(N - N_c) + \mu_{2L}^2(N - N_c)^2]. \tag{18}$$

Since criticality is reached by letting μ_{1L} vanish, the coefficient of the term linear in $(N - N_c)$ vanishes, producing an effective LTP behaviour although the actual critical point is not an LTP. This violation of universality is a consequence of the very special path along which the critical point is approached. In fact, it is only along the trajectory (16) that the proper cancellation of terms leading to (18) can occur. This behaviour is non-generic, because approaching the same critical point along any other path (17) does not hold and the critical behaviour corresponds to a Lifshitz point, with $D^* = 6$.

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